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# TESTING OF PORTFOLIO OPTIMIZATION THROUGH INVESTMENTS IN STOCK MARKET INDICES AND BITCOIN

## Ispitivanje portfolio optimizacije ulaganjem u berzanske indekse i bitkoin

## Abstract

This paper presents an empirical verification of the effectiveness and usefulness of investment diversification using the main stock exchange indices and Bitcoin. The objective is to determine the effects applying the Markowitz portfolio optimization theory, i.e., the advantages of applying the modern portfolio theory for institutional investors. The research offers an answer to the following question: what are the advantages and disadvantages of using Bitcoin in portfolio optimization? The paper contributes to the representation of the reach and limitations of the modern portfolio theory for institutional investors. The conclusion is that rational behaviour of institutional investors requires consideration of portfolio optimization using the Markowitz model, because it is possible to create portfolios which, on the basis of historical returns, provide desired returns alongside certain risks. The methodology includes the analysis of high frequency data, i.e., daily trading data were used. The results indicate that the use of the Markowitz portfolio selection method, with all its limitations, is desirable, possible and applicable, but that it entails serious flaws in the sense of neglecting transaction costs, foreign exchange differences and the real value in the stock market. The results of the research show that Bitcoin is a good source of diversification in a portfolio that contains traditional financial instruments both for the risk-averse investor as well as for those investors who have a greater appetite for risk. The conclusion is that rational behavior of institutional investors requires consideration of investing in Bitcoin using the Markowitz model. However, given the high degree of volatility, investors should be very careful when making decisions about including Bitcoin in the portfolio.

**Keywords:** *portfolio diversification, portfolio optimization, Bitcoin, optimal portfolio, risk, return.*

## Sažetak

Rad se bavi empirijskom provjerom djelotvornosti i korisnosti diverzifikacije ulaganja koristeći glavne berzanske indekse i bitkoin. Cilj istraživanja je da se utvrde efekti primjene Markoviceve portfolio diverzifikacije odnosno optimizacije portfolija, tj. koristi od primjene moderne portfolio teorije za institucionalne investitore. Istraživanje će ponuditi odgovor na pitanje: „Koje su prednosti i nedostaci korištenja bitkoina u optimizaciji portfolija?" Doprinos rada ogleda se kroz predstavljanje dometa i ograničenja moderne portfolio teorije za institucionalne investitore. Zaključak je da racionalno ponašanje institucionalnih investitora nalaže razmatranje optimizacije portfolija upotrebom Markovicevog modela, jer je moguće kreirati portfolije koji na bazi istorijskih prinosa daju željene prinose uz određene rizik. Metodologija podrazumjeva analizu podataka visoke frekvencije, odnosno korišteni su dnevni podaci o trgovanju. Međutim, rezultati primjene značajno odstupaju od očekivanog prinosa. Rezultati pokazuju da je upotreba Markovicevog metoda portfolio selekcije, uz sva ograničenja, poželjna, moguća i primjenljiva, ali da ona ima ozbiljna ograničenja u smislu zanemarivanja transakcionih troškova, kursnih razlika i stvarne trgovine na berzi. Rezultati pokazuju da bitkoin predstavlja dobar izvor diverzifikacije u portfoliju koji sadrži tradicionalne finansijske instrumente kako za investitora koji nije sklon riziku, tako i za one investitore koji imaju veći apetit za rizik. Zaključak je da racionalno ponašanje institucionalnih investitora nalaže razmatranje ulaganja u bitkoin upotrebom Markovicevog modela. Ipak, uzimajući u obzir visoki stepen volatilnosti, investitori treba da budu veoma pažljivi kada donose odluke o uključivanju bitkoina u portfolio.

**Ključne reči:** *diverzifikacija portfolija, optimizacija portfolija, bitkoin, optimalan portfolio, rizik, povrat.*

## Introduction

The possibilities of diversification have always been interesting for investors since diversification diminishes risk exposure and protects investors. Diversification potential enables investors to manage their risk and diminish exposure to risk. A good diversification policy represents a safety net which enables the reduction of investors' risk exposure and prevents a decrease in portfolio value. A well-diversified portfolio more often than not consists of different asset categories with low correlation, whereas highly correlated markets are characterized by a low diversification possibility. The greatest mystery in the sphere of investments is finding the optimal portfolio using available assets. There are a number of studies and mathematical models concerned with portfolio investment strategies. The contemporary portfolio theory aims at finding the optimal model with the best results.

During the years, financial markets have undergone immense changes. One of them is surmounting the obstacles for foreign portfolio flows shifting from one market to another [1] and the emergence of cryptocurrencies as financial assets. This paper will attempt to combine the classic modern portfolio theory with the concept of investing in cryptocurrency with the aim of achieving higher returns with a lower risk exposure.

The research question is: what are the advantages and disadvantages of using Bitcoin in portfolio optimization? The main research hypothesis is that geographical portfolio diversification for institutional investors, alongside all limitations of foreign markets and investors' placements, is desirable, possible and applicable. The supplementary hypothesis is the following: by combining investments in stock market indices and Bitcoin, it is possible to create a portfolio which rejects larger returns and lower risk in relation to a portfolio gained using the same securities but without investing in Bitcoin.

With the aim of testing the hypothesis proposed in this paper, the emphasis was put on the modern portfolio theory. The basic model was developed by Harry Markowitz in 1952 in his paper *Portfolio Selection* (Markowitz, 1952). The most significant contribution of the modern portfolio theory is the formula for the calculation of portfolio variance, i.e., for the calculation of an efficient portfolio diversification. Markowitz's most important assumption regarding investors' behavior within the model is that investors estimate portfolio risk on the basis of the variability of expected returns. Another assumption is that decisions on investments made by investors are mostly based on expected returns and risk. Hence, their utility curves are a function of the expected return and expected variance (or standard deviation) of the return. Also, an important hypothesis is that investors always prefer the highest returns for a certain risk level, i.e., the lowest risk for the same return level.

Financially speaking, a portfolio represents a combination of different asset types, i.e., financial instruments and deposits. Financial assets comprise any form of property with a value which can be exchanged. This can include securities, money in giro accounts, assets in foreign currency, gold and noble metals, cryptocurrency, etc. Two basic motives for portfolio creation are financial gain and risk diversification for the investor. The choice of an appropriate portfolio depends on the expected return rate, the risk of certain securities, correlation (the connection between returns) of certain securities, as well as investors' preferences (a tendency for or aversion to risk). The concept of diversification with the aim to diminish risk is most vividly reflected in the saying "Don't put all your eggs in one basket." When it comes to creating a securities portfolio in the financial market, it takes a lot more analysis and effort than a simple random choice of different securities.

In order to prove the hypothesis, we will observe a fictional investment fund which has two strategies: investing in developed countries and investing in emerging economies. Besides that, we will illustrate the effects of investments for all strategies without limitations in a single position, investments with limitations and investments with the combination of observed assets and Bitcoin.

## Literature review

While classic portfolio theory performed diversification in order to diminish risk, the modern theory introduces a measurement of returns and risk, based on which

it corrects returns on the basis of risk and creates an efficient portfolio. Consequently, the modern theory represents a mathematical and statistical formulation of the diversification concept [2]. The development of investment diversification coincided with the development of portfolio theory [3]. The Markowitz portfolio selection model includes the identification of available risk-return combinations from a set of risky assets, construction of the optimal portfolio of risky assets and, then, the selection of the complete portfolio by combining risk-free assets and optimal risky portfolio [4].

Diversification aims at choosing a number of financial instruments which, observed as a portfolio, have a risk which is lower than the weighted risk average of securities included therein, if returns on securities included in the portfolio do not have a perfect positive correlation. This is possible because returns on financial instruments often take different directions, especially in periods of crisis. Besides, shares from specific sectors can also have a negative correlation, meaning that their prices take opposite directions and all of this therefore provides the opportunity to diminish risk when these securities are found in the portfolio.

The basic motive behind the interest of institutional investors is placement diversification, professional management and the simplicity of capital investments [5]. As financial intermediaries, institutional investors represent competition for traditional banks in the financial market. Institutional investors are specialised financial institutions which gather, or synthesize, small investors' savings by investing it in different investment forms in accordance with defined goals, such as acceptable risk, return maximization and claim maturity [6]. By looking at a wider context, the significance of institutional investors is reflected in raising the efficiency of the financial system [7]. They ensure better investment possibilities and more efficient channels for the allocation of economic resources "both through space and through time" [8]. In relation to that, institutional investors contribute to a larger supply in the market and facilitate the collection of capital for companies listed in the stock market. As a result, capital is cheaper, as well as services, which affects the decrease in fees and other expenses regarding trade and the procurement of capital in the market. Hence, they enrich the structure of the financial sector, i.e., deepen the capital market. At the same time, institutional investors largely contribute to the increase in savings by collecting smaller amounts of capital from individual owners, as well as to the rationalization of the use of capital by placing it in securities [9]. Three types of financial institutions are specifically prominent: pension funds, insurance companies and investment funds. Institutional investors have contributed to the elaboration of investment options for individual investors, competition strenthening in the market and to the balancing of bank-centered financial systems [7].

The term 'emerging market' includes countries characterized by institutional turbulence and a low level of economic development in relation to developed countries. In accordance with this, an emerging market may represent a country, or a market, in which there is an ongoing transition of the political or economic system and in which the economic development is higher than a one-digit percentage at the annual level [10]. Sixty-four emerging economic systems have been identified [11].

The most important advantage of the portfolio theory in relation to the classic theory is that it analyzes individual securities using correlation. Correlation, or the degree of connection between individual securities, is one of the key factors determining the success of a portfolio [12]. On the other hand, a detailed analysis of the portfolio theory and its proposition in every capital market points to the fact that it significantly simplifies the complex world of trading in securities. The first problem, often neglected in a large number of economic models, is ignoring transaction costs. Besides these costs, institutional restrictions on trading are also ignored. The extent to which this assumption limits the practical validity of the theory depends on the impact of transaction costs and institutional limitations on the manner of portfolio creation and trading [13].

Since the emergence of the modern portfolio theory, numerous authors have in different ways attempted to suggest to investors ways to properly choose initial sets of shares to be efficiently diversified in the portfolio. Considering specific conditions in small capital markets, what was set as the primary criterion for the selection of

shares when creating a portfolio is liquidity. The critique boils down to the fact that these costs are insignificant in the course of the initial portfolio creation, but they are important in cases when the portfolio needs to be adapted to the results obtained by applying the Markowitz model.

In accordance with the classic theory of economics, it is assumed that investors mostly aim at maximizing profit. Besides that, it is possible to reassess the assumption of the portfolio theory that all investors have the same information. Namely, it is widely accepted that there are differences in information, i.e., that one contracting party has more information than the other. What emerges here are problems with negative selection and moral hazard. Small investors struggle with obtaining information. On the other hand, this information asymmetry may be reduced by means of intermediaries, such as brokers, who own more information than individual investors. Nonetheless, it is clear that their level of awareness also differs. Namely, brokers' level of awareness depends on the possibility of obtaining additional information from issuers, as well as on the knowledge and skills for the interpretation of information available to the public, for example the impact of macroeconomic indicators or changes in regulations. Modeling results depend on the data to be used for a longer or shorter historical period or to be incorporated in future expectations. Depending on the data used in the model, each investor may have different data, which also leads to different investment decisions. A consequence of the assumption about knowing the same information would be that all investors create the same portfolio, i.e., that possible portfolio combinations are reduced as much as possible. The reason for this is that, according to the theory, all investors have access to the same information which contain the return rates of all securities in the past [13], [9].

Portfolio theory hypothesizes that stock exchange rates are arranged according to normal distribution. However, in reality, stock exchange rates have a large standard deviation which could not be foreseen by normal distribution. Classic economists have the tendency to eliminate all maximum deviations as an anomaly in order to obtain normal data distribution for their analyses. Still, an appropriate analysis needs to include all periods with extreme changes in prices since neglecting such periods would lead to a normal distribution which does not contain significant information. In relation to this, if stock exchange rates varied in accordance with the normal distribution, the stock market breakdown in 1987 with more than twenty standard deviations would have had a probability of one to a billion [14]. Also, an external shock, such as the 2020 pandemic, was almost impossible to consider during 2019.

The possibility of creating a portfolio using domestic securities through the imitation of foreign indices in order to achieve higher returns with no direct foreign exposure was examined [15]. They have determined that investments in foreign markets result in significant diversification advantages.

In 2003, Li, Sarkar and Wang [16] identified significant benefits for international diversification for an investor, headquartered in the US, despite portfolio limitations, especially when it comes to short-term trading, i.e., the sale of securities not yet owned by the investor. On the other hand, in 2004, Kearney and Lucey [17] emphasized reduced diversification benefits in emerging markets as correlations increase over time [18], [19]. Many authors have confirmed these conclusions [20], [21], [22].

At the same time, Berger, Pukthuanthong and Yang [23] showed in 2009 that a correlation between different markets does not have to necessarily indicate the level of integration of the observed markets. Furthermore, Berger et al. [23] use a variance analysis in order to point to benefits arising from international diversification, especially when there are investments in emerging markets. Alongside observing benefits of international investments per se, subsequent papers were also concerned with nuancing international diversification based on style. Estrada [24] examines the advantages of international diversification when applying fundamental indexation and identifies significant benefits for index diversification based on foundations using countries' index funds. Similarly, Eun, Huang and Lai [25] consider international portfolio diversification between 1980 and 1999 by using precisely variance tests. In order to deal with this issue, Fan [10] observes the benefits of international diversification using a sample of indices of large and small capital of G7

countries. The empirical findings of our study suggest that, during the whole period, investors will benefit from diversification through a combination of investments in companies from developed and emerging economies [26]. Li et al. [16] have synthesized research in this area and claimed that when ex post data are observed, there is huge potential for international diversification. On the other hand, the results for ex ante data are questionable due to correlation changes over time. In their paper, they also cite authors who had researched the significance and faster growth of emerging markets and diversification possibilities in these markets [28], [29], [30], [31], [32], [33].

Cryptocurrencies have nowadays penetrated the flows of real economies, and this phenomenon is a point of interest for economists, jurists, IT experts, hackers, central banks and security agencies. One of the reasons for the initial interest of technology and internet supporters and investors is the fact that cryptocurrencies are not subject to control by central banks or government agencies, their value being determined by a multitude of computers. Namely, these are protected from inflation through a mathematical function which makes it impossible for their quantity to exceed a limit determined in advance. A second feature of cryptocurrencies, seen as an advantage, reflects in eliminating intermediaries, which makes transactions cheaper – especially in international payments. What is emphasized as a benefit of cryptocurrencies is the fact that they are based on a decentralized system without the existence of a regulatory authority. On the other hand, it is precisely decentralization, user anonymity and the lack of a regulatory agency what is indicated as the main disadvantage [34]. Using the multivariate dynamic conditional correlation (DCC) model, Chuen, Guo and Wang [35] examined whether the inclusion of ten cryptocurrencies in a traditional portfolio of nine assets would bring additional benefits on risk-adjusted returns. Their findings suggest that the inclusion of the cryptocurrency index provides large improvements in the overall portfolio performance based on mean-variance, which was confirmed by the spanning test employed. Similarly, Ehlers and Gauer [36] employed the Kolmogorov–Smirnov test and variance ratio test (VRT) with heteroscedasticity adjustment to understand the role of five leading cryptocurrencies such as Bitcoin, Ethereum, Ripple, Litecoin, and Dash in a portfolio. They unearthed interesting findings that only Bitcoin and Ripple were shown to provide minimum variance portfolio benefits and volatility exposure, in line with Harry Markowitz's idea on the mean-variance portfolio. Briere, Oosterlinck and Szafarz [37] used weekly data over the period from 2010-2013, and they analyzed a Bitcoin investment from the standpoint of a U.S. investor with a diversified portfolio including both traditional assets (worldwide stocks, bonds, hard currencies) and alternative investments (commodities, hedge funds, real estate). They concluded that, during the observed period, Bitcoin investment had highly distinctive features, including the exceptionally high average return and volatility. Also, they concluded that Bitcoin investments offer significant diversification benefits. Carpenter [38] used a modifiied mean-variance framework and showed that Bitcoin can be a viable diversification tool. His research showed that Bitcoin investment could be skewed by return activity that occurred during a speculative bubble in 2013 [39]. The stated conclusions are in line with Grujić's [22] recommendation that it is rational for financial intermediaries in developing countries to change their business models and adapt them to the accelerated market changes [40].

It may be said that the use of cryptocurrencies in the sense of an alternative to classic money is greatly limited for two reasons. Firstly, although the amount of cryptocurrencties is limited, every individual or group may create, agree on the name of a currency and rules, and use a multitude of other cryptocurrencies. There are already more than a thousand cryptocurrencies. Secondly, despite the limited supply, the price of cryptocurrencies depends on the supply and demand. Supporters of the use of cryptocurrencies point to the limitation in cryptocurrency supply. Namely, in classic currencies the supply is limited by the amount of money as prescribed by central banks. Therefore, on the one hand there is a 'quasi-limitation' of supply, and on the other hand, there is some variability in demand, which points to a significant instability of the central bank. Bitcoin can be split up to the amount of 0.00000001 Bitcoin, i.e., up to a Satoshi. The smallest unit was named Satoshi, after the author (or several authors) of

the original document in which it was initially mentioned [41]. The system enables an unlimited number of everyday transactions but, as the time passes, Bitcoin will also be lost in everyday trading. The nonexistence of a regulatory system and anonymity render Bitcoin suitable for the financing of criminal activities, from money laundering and financing of crime to drug and weapons trafficking and terrorism. Hence, transactions in virtual currencies are public and largely impossible to trace. In this way, a high level of anonymity is ensured for virtual currency users. Precisely such malpractice may affect the closing of the currency trading platform and disabling of access or the use of assets in certain platforms or stock markets. Large changes and constant price growth represented an attractive instrument both for professional investors and for complete amateurs when it comes to investing. At the same time, it was specifically high variability what led professional investors to be careful regarding long-term investments in cryptocurrencies. Unlike other currencies, cryptocurrencies are not supported by the rule of law, but by technology. Transfer is simple, there are no intermediaries and a third party may not prevent or change transactions. Assuming that all legal systems broke down, cryptocurrencies would continue to exist with the existence of the Internet and people ready to use them. Other important features of cryptocurrencies are that ownership is secured by strong cryptography, transactions are visible but the users are anonymous, the person sending cryptocurrencies, as opposed to the receiver, must be connected to the internet and nothing guarantees its value besides the supply and demand.

## Methodology

The modern portfolio theory uses basic statistical categories, such as: variance, standard deviation, correlation and covariance, and categories such as beta and other derived indicators, in order to set up and measure the connection between return and risk in the portfolio. The basic difference between the classic portfolio theory (which dealt with the diversification of different types of assets by combining their different rates of risk and return and performed individual selection of securities on the basis of the analysis of individual shares) and the modern portfolio theory lies in the fact that the contemporary portfolio theory introduces a mathematical and statistical analysis when choosing a portfolio with the aim of creating an optimal portfolio. The Markowitz model is based on several assumptions which are related to investors' behaviour. The most important among these rely on the maximum expected return on the portfolio with a risk level acceptable for them, or alternatively, the reduction of risk to which they are to be exposed alongside a certain assumed level of the expected return on the portfolio.

According to this theory, larger return does not necessarily have to entail higher risk.

The assumptions within the portfolio theory advocated by Markowitz are the following:

- investors consider any investment alternative represented by the distribution of the probability of expected returns in the observed period;
- investors assess portfolio risk on the basis of the variability of the expected portfolio returns;
- investors' decisions are only based on the expected return and risk, their utility curves therefore being a function of the expected returns and the expected variance (or standard deviation) of returns;
- investors maximize the expected utility and their utility curves demonstrate the diminishing marginal wealth utility;
- for a given risk level, investors prefer higher returns as opposed to lower ones and vice versa. For a given level of expected returns, investors prefer lower risk to higher risk.

This paper treats indices as separate securities. In reality, exchange-traded funds replicating the changes in indices are the closest equivalent to that. In order to determine the risk of individual positions being invested in, what is necessary is the standard deviation, or variance, since it measures the extent to which specific amounts, in this case returns, are far from the average. A variance is defined as a square deviation from the average, as indicated by the following calculation pattern:

$$\sigma^2 = \sum_{i=1}^{n} [R_i - E(R_i)]^2 P_i$$

Squaring is advantageous since positive and negative deviations are not mutually effaced. However, squaring leads to large numbers difficult to interpret precisely, therefore resulting in the recommendation to use the variance root, the so-called standard deviation:

$$\sigma = \sqrt{P_i[R_i - E(R_i)]^2}$$

We can see that the correlation between the securities returns, for example A and B, may be expressed through the following pattern:

$$(p_A, p_B) = \frac{\Sigma_{i=1}^{n}(p_A - \mu_A)(p_B - \mu_B)}{\sqrt{\Sigma_{i=1}^{n}(p_A - \mu_A)^2 * \Sigma_{i=1}^{n}(p_B - \mu_B)^2)}}$$

However, for our analysis it is necessary to note the manner in which standard deviation is observed in the portfolio. Because of that, the correlation coefficient needs to be merged with the standard deviation of shares in the portfolio. This yields the so-called covariance calculated using the following pattern:

$$cov_{AB} = \sigma_A \sigma_B \rho_{AB}$$

After defining the share of individual securities in the portfolio, the average return and standard deviation of the portfolio return may be calculated. When calculating the average portfolio return, we simply calculate the weighted average of individual securities' returns [2], p 81).

The expected portfolio return depends on the share of securities in the portfolio. The total portfolio return will be between the return on a security with the lowest expected return and the return on a security with the highest expected return (Markowitz, 1952, pp. 83-84).

The variance of a portfolio with N securities is calculated using the following pattern:

$$Var\,(p) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

This formula is one of the greatest values of portfolio theory. It proves that portfolio risk depends on the securities correlation, or a certain type of property. The contribution of this formula amounts to the suggestion that, when creating a portfolio, what should be analyzed is the extent to which returns on specific asset types are connected. If the correlation exceeds 1, in this case the data have the identical development; on the other hand, if the correlation amounts to -1, the data entail the opposite development (for instance, an increase in the price of instrument A leads to a decrease in the price of instrument B), while a correlation amounting to 0 means that data develop in completely different ways.

Combining securities in the portfolio, if they do not express a perfect positive correlation, affects the reduction of portfolio risk. Therefore, portfolio diversification is a result of combining securities, investments with returns which do not have a perfect positive correlation. Examples of diversified portfolios closely adhere to market returns. In case we invest in a single security, the portfolio risk, incorporated in this single security, then equals its standard deviation. If the number of randomly chosen shares in the portfolio is increased, the total portfolio risk is reduced and this reduction occurs at a declining rate. Briefly, portfolio diversification represents the result of combining assets being invested in, i.e., all securities and other investments with returns which do not have a perfectly positive correlation. Portfolio risk depends on the correlation between returns on the assets in the portfolio.

When creating a risky portfolio, consisting of two types of risky assets, what is most important is the connection between changes in returns on assets. Portfolio risk depends on the correlation between returns on assets in the portfolio. Unsystematic risk more and more slowly approaches zero when new chosen securities are added into the portfolio. As the number of chosen securities to be included in the portfolio increases, the total risk diminishes, but there is still the undiminished part with the increase in the number of securities in the portfolio.

In accordance with the noted subject of analysis, what was observed are the indices of the world's most famous stock exchanges during 2019. Specifically, not all securities quoted in the observed markets were included in the analysis. Such a simplification of the analysis concurs with the remark of a large number of authors that, by including securities above a larger number in the portfolio, the diversification effect is reduced. For each index we have calculated expected returns at the annual level and the matrix of variance and covariance, also at the annual level, with other securities in the portfolio.

The following limitations were set:

- the sum of the percentage share of securities in the portfolio needs to be over 100%,
- the amount of certain securities in the portfolio must not be negative, i.e., we have disregarded the so-called short-term sale and
- a separate calculation is made for each demanded return.

On the basis of the variance-covariance matrix, i.e., on the basis of expected returns and risk of shares, we have determined the shares of securities in the portfolio for which the demanded return will be achieved with the lowest possible risk, or with the lowest portfolio variance.[1]

$$\sigma_P^2 = w^T * \sum w$$

The following pattern is obtained after elaborating this equation:

$$\sigma_P^2 = [w_1\, w_2 \ldots w_i \ldots w_n] \begin{bmatrix} \sigma_{11}\, \sigma_{12} \ldots \sigma_{1j} \ldots \sigma_{1n} \\ \sigma_{21}\, \sigma_{22} \ldots \sigma_{2j} \ldots \sigma_{2n} \\ \ldots \ldots \ldots \ldots \ldots \ldots \\ \sigma_{i1}\, \sigma_{i2} \ldots \sigma_{ij} \ldots \sigma_{in} \\ \ldots \ldots \ldots \ldots \ldots \ldots \\ \sigma_{n1}\, \sigma_{n2} \ldots \sigma_{nj} \ldots \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \ldots \\ w_i \\ \ldots \\ w_n \end{bmatrix}$$

The expected return on the obtained MEF curve (*Markowitz efficient frontier*) varies from the largest expected return on a share (if we invested 100% of the amount only in a security with the largest share) to the lowest expected return on a share (if we invested 100% of the amount only in a share yielding the lowest return).

1 With the aim of a faster and more accurate calculation, portfolio optimization was derived using the Solver subprogram. This is a free add-in of the Microsoft Excel program package.

## Empirical data and analysis

The data were collected from the stock exchanges of the observed countries. The countries were divided according to the criterion used by Hoskisson, Eden, Lau and Wright [11] for developed and emerging economies and they are presented in Table 1.

Variance-covariance matrices were created for both combinations (Table 4, Table 5).

## Results and discussion

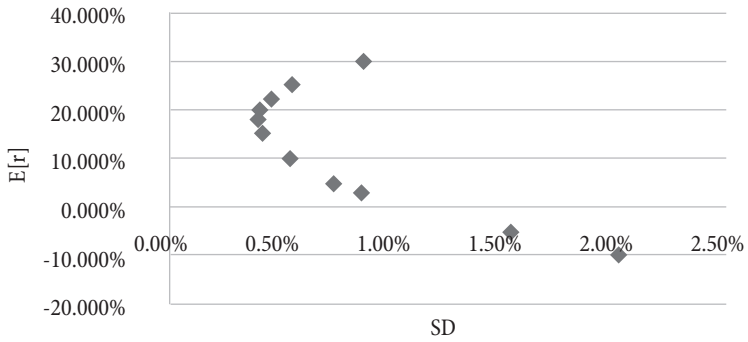
The obtained results adhere to conclusions provided by Li et al. (2003), Kearney and Lucey (2004), Switzer and Tahaoglu (2015), Fan (2008), Estrada (2008), Eun et al. (2008), Berger et al. (2013). The paper proves that the use of the Markowitz method of portfolio selection in the shares in emerging markets is, with all its limitations, possible, i.e., it is desirable and applicable.

By applying the diversification model, or portfolio optimization, different investment combinations were obtained. An investor has the possiblity to optimize the portfolio by investing only in developed economies (Figure 1) or only in emerging economies (Figure 2).

Similarly, by applying the described diversification model, i.e., portfolio optimization, the investor has the possibility to optimize the portfolio by also considering changes in the value of Bitcoin (Figure 3) or only in emerging economies (Figure 2).

If we observe the results obtained for expected returns of 5%, 10%, 15%, 20%, 25% and 30%, we can observe that, according to the model, by aiming for the

**Figure 1: The Markowitz efficient frontier for developed economies**




Source: Authors' calculations.

**Table 1: Presentation of observed countries and analyzed indices**

| Number | Country | Index | Emerging economy | 2020 | 2019 | 2018 |
|---|---|---|---|---|---|---|
| 1 | United States | DJIA | No | 30,409.56 | 28,538.44 | 23,327.46 |
| 3 | Japan | Nikkei 225 | No | 26,854.03 | 23,656.62 | 20,014.77 |
| 4 | Germany | DAX 30 | No | 13,718.78 | 13,249.01 | 10,558.96 |
| 5 | France | CAC 40 | No | 5,599.41 | 5,978.06 | 4,730.69 |
| 6 | United Kingdom | GSPTSE | No | 17,545.81 | 17,063.43 | 14,322.86 |
| 8 | Italy | FTSEI40 | No | 2,165.00 | 2,289.40 | 1,782.00 |
| 10 | Canada | GSPTSE | No | 17,545.81 | 17,063.43 | 14,322.86 |
| 11 | Australia | AXJO | No | 6,587.10 | 6,684.10 | 5,646.40 |
| 12 | Spain | IBEX35 | no | 8,154.40 | 9,549.20 | 8,539.90 |
| 17 | Netherlands | AEX | no | 628.06 | 604.58 | 487.88 |
| 18 | Switzerland | SMI | no | 10,703.51 | 10,616.94 | 8,429.30 |
| 19 | Sweden | OMXS30 | no | 1,874.74 | 1,771.85 | 1,408.74 |
| 20 | Norway | OSE | no | 973.97 | 931.45 | 799.46 |
| 21 | Belgium | BFX | no | 3,663.06 | 3,955.83 | 3,243.63 |
| 22 | Austria | ATX | no | 2,780.44 | 3,186.94 | 2,745.78 |
| 25 | Denmark | OMXC20 | no | 1,465.17 | 1,135.79 | 1,135.79 |
| 28 | Finland | OMXHPI | no | 27,147.11 | 28,189.75 | 25,845.70 |
| 29 | Greece | ATG | no | 10,872.05 | 9,874.66 | 8,709.58 |
| 31 | Portugal | PSI20 | no | 1,567.46 | 1,616.70 | 1,333.18 |
| 32 | Ireland | ISEQ 20 | no | 4,921.78 | 5,214.14 | 4,731.47 |
| 33 | New Zealand | NZX 50 | no | 1,318.32 | 1,196.30 | 909.82 |
| 2 | China | SSEC | yes | 3,414.45 | 3,050.12 | 2,493.90 |
| 7 | Brazil | BVSP | yes | 119,017.24 | 115,645.34 | 87,887.26 |
| 9 | India | BSESN | yes | 47,746.22 | 41,253.74 | 36,068.33 |
| 13 | Mexico | S&P_BMV IPC | yes | 44,693.96 | 43,541.02 | 41,640.27 |
| 14 | South Korea | KOSPI | yes | 2,873.47 | 2,197.67 | 2,041.04 |
| 15 | Indonesia | JKSE | yes | 5,979.07 | 6,299.54 | 6,194.50 |
| 16 | Turkey | BIST100 | yes | 1,479.91 | 1,144.25 | 912.70 |
| 23 | South Africa | SAT40 | yes | 54,615.33 | 50,816.05 | 46,726.59 |
| 24 | Thailand | SETI | yes | 1,449.35 | 1,579.84 | 1,563.88 |
| 26 | Chile | SPIPSA | yes | 327.43 | 372.16 | 342.92 |
| 27 | Hong Kong | HSI | yes | 4,177.22 | 4,669.85 | 5,105.43 |
| 30 | Israel | TA125 | yes | 1,567.46 | 1,616.70 | 1,333.18 |
| 34 | Croatia | CROBEX10 | yes | 1,739.29 | 2,017.43 | 1,748.81 |
| 35 | Slovenia | SBITOP | yes | 900.85 | 925.86 | 804.26 |
| 36 | Serbia | BELEX15 | yes | 746.57 | 801.69 | 761.69 |
| 37 | Bosnia and Herzegovina | BIRS | yes | 576.94 | 618.31 | 565.48 |
| 38 | Bosnia and Herzegovina | SASX30 | yes | 1,428.43 | 1,288.46 | 1,135.79 |
| 39 | Bitcoin | BTC | no | 28,840.95 | 7,193.60 | 3,742.70 |

Source: Authors' calculations.

**Table 2: Illustration of expected returns for developed countries in 2019 and returns achieved in 2020**

| Country | United States | Japan | Germany | France | United Kingdom | Italy | Canada | Spain | Netherlands | Switzerland | Sweden | Norway | Belgium | Austria | Finland | Greece | Portugal | Ireland | New Zealand |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Expected return | 22.24% | 20.93% | 25.22% | 27.48% | 18.93% | 23.72% | 18.93% | 11.69% | 24.25% | 25.41% | 26.03% | 15.94% | 23.08% | 15.02% | 12.21% | -15.42% | 9.98% | 31.25% | 10.38% |
| Return achieved in 2020 | 6.02% | 18.27% | 2.49% | -8.11% | 1.95% | 12.57% | 1.95% | -16.69% | 1.92% | 0.03% | 3.66% | 3.47% | -9.83% | -13.90% | 8.49% | -13.08% | -6.98% | 5.86% | 13.52% |

Source: Authors' calculations.

**Table 3: Illustration of expected returns for emerging economies in 2019 and returns achieved in 2020**

| Country | China | Brazil | India | Mexico | South Korea | Indonesia | Turkey | Taiwan | Thailand | Singapore | Hong Kong | Israel | Croatia | Slovenia | Serbia | Bosnia and Herzegovina | Bosnia and Herzegovina |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Expected return | 23.72% | 27.07% | 14.94% | 3.00% | 9.34% | 1.92% | 28.76% | 30.99% | 0.89% | 9.53% | 12.17% | 21.17% | 16.66% | 15.01% | 9.55% | 9.30% | 13.44% |
| Return achieved in 2020 | 12.57% | 0.37% | 14.71% | -0.83% | 32.10% | -4.85% | 27.38% | 27.28% | -9.18% | -13.59% | -4.60% | -3.90% | -14.29% | -3.24% | -6.21% | -6.76% | 10.88% |

Source: Authors' calculations.

## Table 4: Variance-covariance matrix for developed countries

| | United States | Japan | Germany | France | United Kingdom | Italy | Canada | Spain | Netherlands | Switzerland | Sweden | Norway | Belgium | Austria | Finland | Greece | Portugal | Ireland | New Zealand |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| United States | 0.000061 | 0.000007 | 0.000047 | 0.000047 | 0.000026 | 0.000046 | 0.000026 | 0.000038 | 0.000041 | 0.000031 | 0.000043 | 0.000035 | 0.000042 | 0.000041 | 0.000039 | -0.000006 | 0.000036 | 0.000043 | 0.000002 |
| Japan | 0.000007 | 0.000071 | 0.000014 | 0.000013 | 0.000006 | 0.000008 | 0.000006 | 0.000012 | 0.000016 | 0.000007 | 0.000020 | 0.000018 | 0.000021 | 0.000019 | 0.000015 | -0.000026 | 0.000013 | 0.000019 | -0.000002 |
| Germany | 0.000047 | 0.000014 | 0.000077 | 0.000066 | 0.000025 | 0.000065 | 0.000025 | 0.000058 | 0.000057 | 0.000043 | 0.000064 | 0.000048 | 0.000062 | 0.000057 | 0.000053 | -0.000013 | 0.000048 | 0.000065 | 0.000004 |
| France | 0.000047 | 0.000013 | 0.000066 | 0.000070 | 0.000025 | 0.000063 | 0.000025 | 0.000054 | 0.000057 | 0.000044 | 0.000061 | 0.000047 | 0.000060 | 0.000052 | 0.000049 | -0.000016 | 0.000045 | 0.000060 | 0.000004 |
| United Kingdom | 0.000026 | 0.000006 | 0.000025 | 0.000025 | 0.000021 | 0.000025 | 0.000021 | 0.000019 | 0.000022 | 0.000017 | 0.000023 | 0.000019 | 0.000023 | 0.000020 | 0.000020 | 0.000005 | 0.000019 | 0.000019 | 0.000001 |
| Italy | 0.000046 | 0.000008 | 0.000065 | 0.000063 | 0.000025 | 0.000085 | 0.000025 | 0.000057 | 0.000053 | 0.000041 | 0.000058 | 0.000048 | 0.000059 | 0.000056 | 0.000050 | -0.000021 | 0.000048 | 0.000060 | -0.000001 |
| Canada | 0.000026 | 0.000006 | 0.000025 | 0.000025 | 0.000021 | 0.000025 | 0.000021 | 0.000019 | 0.000022 | 0.000017 | 0.000023 | 0.000019 | 0.000023 | 0.000020 | 0.000020 | 0.000005 | 0.000019 | 0.000019 | 0.000001 |
| Spain | 0.000038 | 0.000012 | 0.000058 | 0.000054 | 0.000019 | 0.000057 | 0.000019 | 0.000061 | 0.000047 | 0.000035 | 0.000050 | 0.000040 | 0.000053 | 0.000047 | 0.000044 | -0.000028 | 0.000042 | 0.000056 | -0.000002 |
| Netherlands | 0.000041 | 0.000016 | 0.000057 | 0.000057 | 0.000022 | 0.000053 | 0.000022 | 0.000047 | 0.000055 | 0.000039 | 0.000052 | 0.000042 | 0.000053 | 0.000045 | 0.000042 | -0.000015 | 0.000040 | 0.000052 | 0.000001 |
| Switzerland | 0.000031 | 0.000007 | 0.000043 | 0.000044 | 0.000017 | 0.000041 | 0.000017 | 0.000035 | 0.000039 | 0.000044 | 0.000038 | 0.000035 | 0.000039 | 0.000033 | 0.000033 | -0.000012 | 0.000030 | 0.000039 | 0.000003 |
| Sweden | 0.000043 | 0.000020 | 0.000064 | 0.000061 | 0.000023 | 0.000058 | 0.000023 | 0.000050 | 0.000052 | 0.000038 | 0.000076 | 0.000050 | 0.000057 | 0.000055 | 0.000056 | 0.000003 | 0.000046 | 0.000057 | 0.000010 |
| Norway | 0.000035 | 0.000018 | 0.000048 | 0.000047 | 0.000019 | 0.000048 | 0.000019 | 0.000040 | 0.000042 | 0.000035 | 0.000050 | 0.000066 | 0.000044 | 0.000046 | 0.000044 | 0.000005 | 0.000037 | 0.000043 | 0.000009 |
| Belgium | 0.000042 | 0.000021 | 0.000062 | 0.000060 | 0.000023 | 0.000059 | 0.000023 | 0.000053 | 0.000053 | 0.000039 | 0.000057 | 0.000044 | 0.000071 | 0.000055 | 0.000051 | -0.000018 | 0.000047 | 0.000061 | 0.000001 |
| Austria | 0.000041 | 0.000019 | 0.000057 | 0.000052 | 0.000020 | 0.000056 | 0.000020 | 0.000047 | 0.000045 | 0.000033 | 0.000055 | 0.000046 | 0.000055 | 0.000075 | 0.000053 | -0.000020 | 0.000047 | 0.000059 | 0.000000 |
| Finland | 0.000039 | 0.000015 | 0.000053 | 0.000049 | 0.000020 | 0.000050 | 0.000020 | 0.000044 | 0.000042 | 0.000033 | 0.000056 | 0.000044 | 0.000051 | 0.000053 | 0.000067 | -0.000011 | 0.000043 | 0.000052 | 0.000005 |
| Greece | -0.000006 | -0.000026 | -0.000013 | -0.000016 | 0.000005 | -0.000021 | 0.000005 | -0.000028 | -0.000015 | -0.000012 | 0.000003 | 0.000005 | -0.000018 | -0.000020 | -0.000011 | 0.000661 | -0.000011 | -0.000039 | 0.000088 |
| Portugal | 0.000036 | 0.000013 | 0.000048 | 0.000045 | 0.000019 | 0.000048 | 0.000019 | 0.000042 | 0.000040 | 0.000030 | 0.000046 | 0.000037 | 0.000047 | 0.000047 | 0.000043 | -0.000011 | 0.000058 | 0.000047 | -0.000002 |
| Ireland | 0.000043 | 0.000019 | 0.000065 | 0.000060 | 0.000019 | 0.000060 | 0.000019 | 0.000056 | 0.000052 | 0.000039 | 0.000057 | 0.000043 | 0.000061 | 0.000059 | 0.000052 | -0.000039 | 0.000047 | 0.000097 | 0.000000 |
| New Zealand | 0.000002 | -0.000002 | 0.000004 | 0.000004 | 0.000001 | -0.000001 | 0.000001 | -0.000002 | 0.000001 | 0.000003 | 0.000010 | 0.000009 | 0.000001 | 0.000000 | 0.000005 | 0.000088 | -0.000002 | 0.000000 | 0.000149 |

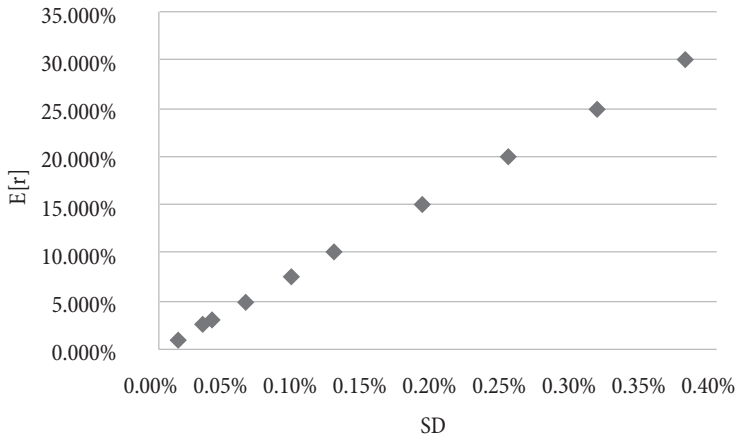
Source: Authors' calculations.

## Table 5: Variance-covariance matrix for emerging economies

| | China | Brazil | India | Mexico | South Korea | Indonesia | Turkey | Taiwan | Thailand | Singapore | Hong Kong | Israel | Croatia | Slovenia | Serbia | Bosnia and Herzegovina | Bosnia and Herzegovina |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| China | 0.00012090 | 0.00000091 | 0.00001636 | 0.00000971 | 0.00003389 | 0.00001803 | 0.00001828 | 0.00003373 | 0.00001203 | 0.00003506 | 0.00001332 | 0.00001230 | 0.00000423 | 0.00000033 | 0.00000121 | 0.00000630 | 0.00000036 |
| Brazil | 0.00000091 | 0.00012373 | 0.00000113 | 0.00003325 | 0.00000840 | 0.00001043 | 0.00001897 | 0.00001401 | 0.00001076 | 0.00000712 | 0.00003037 | 0.00001188 | -0.00000373 | 0.00000150 | -0.00000312 | -0.00000692 | 0.00000139 |
| India | 0.00001636 | 0.00000113 | 0.00007255 | 0.00001092 | 0.00001437 | 0.00001120 | 0.00001419 | 0.00000924 | 0.00000671 | 0.00000936 | 0.00000475 | 0.00000859 | 0.00000284 | -0.00000366 | 0.00000655 | 0.00000092 | 0.00000094 |
| Mexico | 0.00000971 | 0.00003325 | 0.00001092 | 0.00006603 | 0.00001061 | 0.00000795 | 0.00001283 | 0.00002235 | 0.00000870 | 0.00000914 | 0.00002603 | 0.00000856 | -0.00000097 | 0.00000071 | 0.00000308 | -0.00000833 | 0.00000101 |
| South Korea | 0.00003389 | 0.00000840 | 0.00001437 | 0.00001061 | 0.00006045 | 0.00001786 | 0.00001361 | 0.00002678 | 0.00001463 | 0.00002712 | 0.00001700 | 0.00001094 | 0.00000273 | 0.00000185 | -0.00000125 | 0.00000347 | -0.00000026 |
| Indonesia | 0.00001803 | 0.00001043 | 0.00001120 | 0.00000795 | 0.00001786 | 0.00004695 | 0.00001107 | 0.00000846 | 0.00000804 | 0.00001506 | 0.00001530 | 0.00000259 | 0.00000208 | 0.00000074 | 0.00000120 | 0.00000886 | -0.00000146 |
| Turkey | 0.00001828 | 0.00001897 | 0.00001419 | 0.00001283 | 0.00001361 | 0.00001107 | 0.00016442 | 0.00002061 | 0.00000909 | 0.00001796 | 0.00002785 | 0.00000710 | 0.00000023 | 0.00000816 | 0.00001350 | -0.00000220 | -0.00000401 |
| Taiwan | 0.00003373 | 0.00001401 | 0.00000924 | 0.00002235 | 0.00002678 | 0.00000846 | 0.00002061 | 0.00007269 | 0.00001317 | 0.00002268 | 0.00001947 | 0.00001691 | 0.00000369 | 0.00000152 | -0.00000270 | -0.00000021 | 0.00000046 |
| Thailand | 0.00001203 | 0.00001076 | 0.00000671 | 0.00000870 | 0.00001463 | 0.00000804 | 0.00000909 | 0.00001317 | 0.00003325 | 0.00001324 | 0.00001044 | 0.00000738 | 0.00000008 | -0.00000049 | 0.00000340 | -0.00000001 | -0.00000051 |
| Singapore | 0.00003506 | 0.00000712 | 0.00000936 | 0.00000914 | 0.00002712 | 0.00001506 | 0.00001796 | 0.00002268 | 0.00001324 | 0.00004221 | 0.00001203 | 0.00001074 | 0.00000204 | 0.00000240 | 0.00000039 | 0.00000188 | -0.00000085 |
| Hong Kong | 0.00001332 | 0.00003037 | 0.00000475 | 0.00002603 | 0.00001700 | 0.00001530 | 0.00002785 | 0.00001947 | 0.00001044 | 0.00001203 | 0.00010757 | 0.00001266 | 0.00000417 | 0.00000322 | 0.00000022 | 0.00000187 | 0.00000020 |
| Israel | 0.00001230 | 0.00001188 | 0.00000859 | 0.00000856 | 0.00001094 | 0.00000259 | 0.00000710 | 0.00001691 | 0.00000738 | 0.00001074 | 0.00001266 | 0.00003563 | 0.00000131 | -0.00000044 | 0.00000478 | 0.00000177 | -0.00000177 |
| Croatia | 0.00000423 | -0.00000373 | 0.00000284 | -0.00000097 | 0.00000273 | 0.00000208 | 0.00000023 | 0.00000369 | 0.00000008 | 0.00000204 | 0.00000417 | 0.00000131 | 0.00001905 | 0.00000174 | -0.00000104 | -0.00000156 | -0.00000030 |
| Slovenia | 0.00000033 | 0.00000150 | -0.00000366 | 0.00000071 | 0.00000185 | 0.00000074 | 0.00000816 | 0.00000152 | -0.00000049 | 0.00000240 | 0.00000322 | -0.00000044 | 0.00000174 | 0.00001838 | 0.00000228 | 0.00000179 | -0.00000034 |
| Serbia | 0.00000121 | -0.00000312 | 0.00000655 | 0.00000308 | -0.00000125 | 0.00000120 | 0.00001350 | -0.00000270 | 0.00000340 | 0.00000039 | 0.00000022 | 0.00000478 | -0.00000104 | 0.00000228 | 0.00004686 | -0.00000281 | -0.00000125 |
| Bosnia and Herzegovina | 0.00000630 | -0.00000692 | 0.00000092 | -0.00000833 | 0.00000347 | 0.00000886 | -0.00000220 | -0.00000021 | -0.00000001 | 0.00000188 | 0.00000187 | 0.00000177 | -0.00000156 | 0.00000179 | -0.00000281 | 0.00009693 | -0.00000088 |
| Bosnia and Herzegovina | 0.00000036 | 0.00000139 | 0.00000094 | 0.00000101 | -0.00000026 | -0.00000146 | -0.00000401 | 0.00000046 | -0.00000051 | -0.00000085 | 0.00000020 | -0.00000177 | -0.00000030 | -0.00000034 | -0.00000125 | -0.00000088 | 0.00001428 |

Source: Authors' calculations.

## Figure 2: The Markowitz efficient frontier for emerging economies

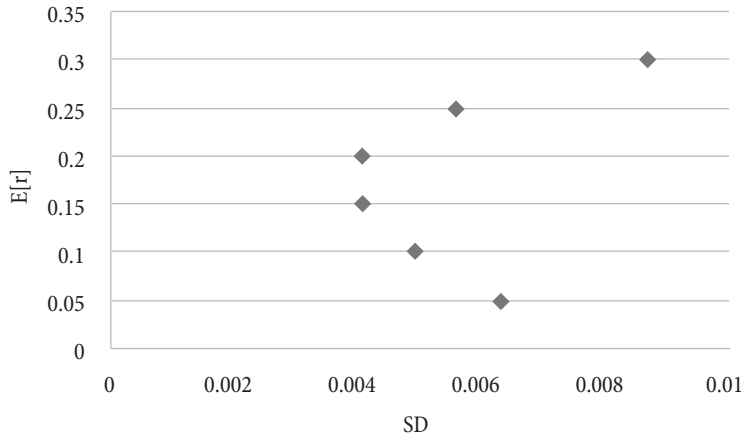



Source: Authors' calculations.

same returns, it is possible in emerging economies to achieve a lower standard deviation, i.e., the investor is exposed to lower risk. It is interesting that by introducing Bitcoin, standard deviation in developed economies is truly reduced to a specific desired return. On the other hand, combining Bitcoin and investment in shares in emerging markets barely differs from the creation of a portfolio with no Bitcoin. Still, investing in Bitcoin in 2020 would have proven to be a very wise decision. Portfolios obtained through diversification based on data for 2019 would successively yield lower returns in relation to those expected. However, if one also invests in Bitcoin, returns are significantly higher than expected (Table 6).

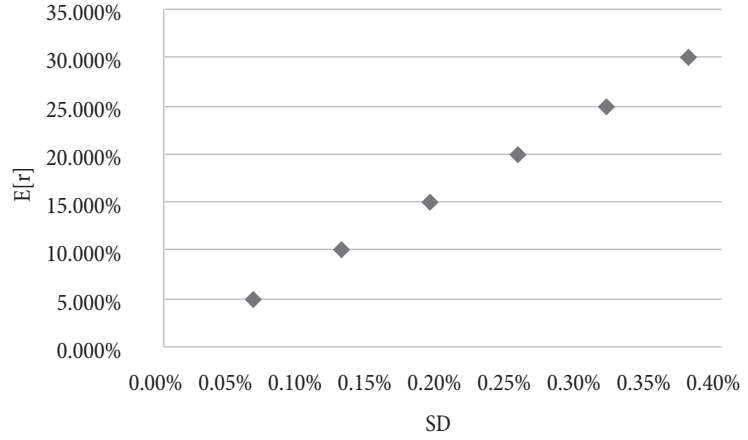
Using minor adjustments in the original formula, it is possible to adjust the variance value to the maximum, minimum or even precisely determined value. In other

**Figure 3: The Markowitz efficient frontier for emerging economies in combination with BTC**




Source: Authors' calculations.

**Figure 4: The Markowitz efficient frontier for emerging economies**




Source: Authors' calculations.

**Table 6: Results of optimization in developed countries**

| Illustration | Standard deviation | | | | Achieved returns in 2020 | | | |
|---|---|---|---|---|---|---|---|---|
| Expected returns | Developed economies | Emerging economies | Developed economies with BTC | Emerging economies with BTC | Developed economies | Emerging economies | Developed economies with BTC | Emerging economies with BTC |
| 5% | 0.736% | 0.063% | 0.631% | 0.063% | -8.21% | 0.37% | 15.02% | 4.33% |
| 10% | 0.540% | 0.126% | 0.491% | 0.127% | -3.12% | 0.63% | 33.40% | 8.89% |
| 15% | 0.421% | 0.188% | 0.407% | 0.190% | 2.39% | 0.94% | 51.75% | 14.64% |
| 20% | 0.406% | 0.251% | 0.406% | 0.253% | 5.34% | 1.09% | 60.60% | 17.81% |
| 25% | 0.556% | 0.314% | 0.556% | 0.317% | 4.90% | 1.41% | 60.47% | 22.55% |
| 30% | 0.868% | 0.377% | 0.868% | 0.377% | 4.62% | 1.72% | 4.62% | 26.90% |

Source: The authors' calculations.

words, it is possible to create different combinations of securities in the portfolio with different risk levels. Besides, it is also possible to adjust certain limitations of exposures towards a certain security, country or index. Furthermore, by setting limitations, both the highest possible gain and the lowest possible gain are diminished because the share of the security bearing extreme returns (negative or positive) is limited. This limitation also affects the variance, or standard deviation of the portfolio.

The portfolio theory, alongside the lack of assumption about the investors' rationality, is flawed in that it does not consider the individual function of an investor's utility and their individual attitude toward risk. Certainly, there are always investors who are ready to take over a higher risk. Another limitation of the modern portfolio theory is the fact that, when choosing the optimal portfolio, neither transaction costs nor foreign exchange differences are considered.

The biggest flaw of this approach is ignoring transaction costs and foreign exchange differences.

## Conclusion

The paper assumes that all institutional investors in the Western Balkans have similar problems. In financial literature, a portfolio is often defined as a collection of two or more securities of different types and features (money market instruments, capital market instruments, financial derivatives, etc.) or other assets (gold and silver) owned by an investor in the financial market. Historically, the portfolio theory went through its two main stages: traditional or classic portfolio theory and the modern portfolio theory.

The current paper differentiates between achieved and expected returns. Achieved returns when investing in a security represent historical returns which show us how much we have earned in the past on account of owning a security, while expected income relates to the future and indicates investors' expectations, or how much an investor should earn in the future based on ownership of a security. Hence, expected returns on a portfolio represent the weighted average of all expected returns on financial instruments constituting the portfolio. The weighting factor is the share of all financial instruments in the portfolio, where the sum of all weighting factors always equals number one.

Unlike expected returns, risk (standard deviation) of a portfolio does not represent the weighted average of standard deviations of securities in the portfolio because in that case the connection between two securities would be neglected. The examination of the direction of changes of two securities is determined by calculating covariance. Through the standardization of covariance, we obtain a correlation coefficient which represents a statistical model indicating the direction in which two securities change and the strength of connection between them.

The paper examines the reach and limitations of the application of portfolio theory in portfolio diversification for institutional investors in emerging financial markets, whereby the research results are encouraging. By using the Markowitz portfolio selection method, the effect of diversification in the observed markets was shown. The results indicate that the use of the Markowitz portfolio selection method with securities in emerging markets is, with all its limitations, desirable, possible and applicable, which proves the accuracy of the hypothesis set in introductory remarks.

However, the obtained results prove that portfolio risk is not significantly reduced through diversification, i.e., the sum of systematic market risk and unsystematic specific risk, since an investor is in certain cases significantly exposed to the risk of a single security.

Despite its revolutionary success in the theoretical domain, the portfolio optimization model in reality demands certain improvements in order for results to be applicable in real investments. A detailed analysis of the modern portfolio theory and its propositions in each capital market indicates that this theory significantly simplifies trading in securities. One of the biggest problems is ignoring transaction costs. Also, this theory assumes that all investors are rational and that they mostly aim at maximizing profit. Besides, it is possible to question the proposition that all investors have the same information. When considering all the limitations, diversification results will also depend on the set of data used for the creation

of the model – whether a longer or shorter series of data will be used or future expectations will be incorporated into modeling.

Emerging capital markets are characterized by extremely low level of share liquidity, which creates an illusion of a negative correlation, further leading to the creation of suboptimal portfolios. Therefore, share liquidity should be the primary criterion for the selection of shares in the portfolio.

One of the most significant critiques also found in connection with the modern portfolio theory is that in financial crises, correlation coefficients converge toward one, whereby diversification advantages disappear and portfolio risk equals the simple weighted sum of the risk of individual securities in the portfolio. Even in developed markets such as the one in New York, in situations of significant financial shocks, there may be some relatively sudden changes in correlation coefficients. Still, the most important recommendation for the creation of portfolio in emerging economies results from the fact that securities liquidity is relatively low in such markets.

What should be considered regarding capital markets in developing countries characterized by low liquidity is a low free float, i.e., a small percentage of shares is freely traded. This means that the investors, when they want to sell the shares they own, might not be able to do so due to a lack of demand in the market. Hence, the modern portfolio theory and diversification are not to be dismissed or understood as the only or the best way of managing risk.

Further research may be concerned with testing models in different time periods. For instance, data from three years ago could be tested within the last year. For example, a certain virtual portfolio might be created on the basis of data from 2014 to 2017 and the obtained portfolio may be tested against data from 2017 until today. Additionally, future research might be directed toward introducing different models of transaction costs and the analysis of the impact of different criteria on the choice of shares in the portfolio, as well as the different length of the time series on the basis of which estimates are obtained. Also, subsequent research might deal with the advancement of optimization in such a way as to introduce the maximum number of transactions and costs for the optimization of portfolio in developing markets, which will simplify decision-making regarding later trading for institutional investors. Finally, research may be directed toward the comparison of the main indices in regional stock markets with the aim to explore whether there is a statistically significant correlation between returns on different indices.

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